MTH 520/622: Introduction to hyperbolic geometry

Practice Assignment III

1. Given non-negative real numbers α_i, for $1 \leq i \leq n$ with $\sum_{i=1}^{n} \alpha_i < (n-2)\pi$, does there exists a hyperbolic n-gon with internal angles α_i?

2. Prove the formulas in 3.4 (xii) (a) - (d) of the lesson plan, using the following hints:

   (a) Consider the hyperbolic line which meets the horizontal radius at a distance a from $O \in \mathbb{D}$. Show that this line is the arc of a circle of Euclidean radius $(\sin ha)^{-1}$.

   Put a vertex B with angle β at $O \in \mathbb{D}$ so that the vertex C is at distance a from O along the horizontal diameter. Let X be the centre of the Euclidean circle through A and orthogonal to $\partial\,\mathbb{D}$. Now use Euclidean cosine formula in triangle OAX to prove that

   $$\cos\beta = \tanh a / \tanh c.$$

   Finally, use hyperbolic Pythagoras Theorem to prove the other identities.

   (b) Hint: Drop a perpendicular from vertex to the opposite side and proceed.

   (c) & (d) Hint: Let the perpendicular from C to AB meet AB in D. If $|CD| = h$ and the angle C in triangle CAD is γ_1, then verify first that

   $$\sin\gamma_1 \cosh h = \cos\alpha.$$